Ultimate Strength Design of Reinforced Concrete Members Under Torsion

by *Hiroshi Matsushima**

1. Introduction

In this paper the author analyzed strength and deformation of the reinforced concrete members under combined bending and torsion, and proposed the design method for such members from the standpoint of the ultimate strength design.

Up to now, torsion has been neglected usually in the design of the concrete structures which is considered as the secondary load. But in the recent new structures, torsion becomes sometimes the main load. For torsion, stiffness of the reinforced concrete member suddenly drops just after initial cracking, and the margin of strength from cracking to failure is usually very small and failure behavior is violent.

First, the past researches about torsion of the structural concrete are summaried. Then, the cracking and failure mechanism of the concrete members under torsion are analyzed, and the design methods are proposed based on these analysis. The member which is the element of the structures is often under eccentric torsion and warping of the member's cross section is restrained, so the effects of these are also considered.

The summary of the past researches is as follows. The study of torsion for the plain concrete members started from application of the classic torsional theories to the concrete members. The elastic torsion theory is St. Venant's one or L. Plandtle's Membrane Analogy using analogical method. The other hand, the plastic theory is A. Nadai's Sandheap Analogy. As the diffrent theory from them, there is Skew Bending Theory proposed by T. T. C, Hsu[1].

For the reinforced concrete members under torsion and combined bending and torsion, three methods have been used for analysis. Truss analogy which has been applied for the first time to torsion of the concrete members by Raush is adapted now by the European Code (CEB-FIP, 1970)[2] and Canadian Code. Limit equilibrium method by N. N. Lessig and et. al[3] is adapted in the Russian Code and the Now Australian Code. T. T. C. Hsu and others proposed the experimental design formulas[4] which are used in the American Code (ACI 318-71)[5]. Even in the former two analytical studies, only the equilibrium condition of forces is considered and compatibility condition of strain is neglected. Moreover, only strength of the members is emphasized and deformation is despised, so there are many problems to be solved in the past researches from the view point of ultimate strength design.

The author's design method based on cracking strength is for the serviceability limit state. Drop of torsional stiffness caused by initial cracking is remarkable and margin of strength from cracking to failure is small, so it is often proper to design for torsion based on cracking strengh. The analysis of cracking strength is based on "Modified Membrane Analogy" and contribution of steel is estimated by its conversion area.

In this paper, failure mechanism is analyzed using "Skew Bending Concept". Namely, the hypothetical member is set up which has the failure surface of the member under combined bending and torsion. These moments acting on the actual member are translated to the loads acting on this hypothetical member. From the equilibrium and copatibility conditions the general expressions for strength and deformation of the member are derived. Design formulas are obtained by givening the yield and failure conditions to these general expressions.

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To estimate the deformation of the member, length of the torsional plastic-hinge is considered. Many charts for design are prepared and tests confirmed these proposed design formulas.

2. Design method for cracking strength[6)]

2.1 Notations

ε_e, ε_p: elastic and plastic strain due to torsion, respectively

$\varphi_{1\sim3}$: torsional constant of elastic theory (St. Venant's or Membrane Analogy), plastic theory (A. Nadal's Sandheap Analogy) and elasto-plastic theory (Modified-Membrance Analogy), respectively

A_c: cross sectional area of concrete

A_s: cross sectional area of longitudinal bars

A_t: equivalent sectional area for torsion

M_t: torsional moment

M_{tc}: torsional cracking moment

a_{sv}: area of one leg of stirrup

b, h: length of shorter and longer eide of rectangular cross section, respectively

$n=E_s/E_c$

$r=a_{sv}/bs$: stirrup ratio

s: spacing of stirrups

2.2 Influences of torsional crack

Torsional crack has larger influences on strength and deformation of the reinforced concrete members than the one of bending moment.

2.2.1 Influence on strengh

Experiments show that margin of strength between initial cracking and failure is smaller than that of bending as shown in **Table 2-1**. Such a tendency is remarkable in pure torsion. This table also shows that reinforcement by only longitudinal bars has little effect and it is necessary to use the longitudinal bars and stirrups.

2.2.2 Influence on deformation

As shown in **Table 2-3**, torsional stiffness drops just after intital cracking to about 10% of non-cracking member's. This is the characteristic of torsion and is noteworthy problem in design.

2.3 Analysis

To estimate the torsional stress there were two methods in the past, one was elastic and the other was plastic theory. The author proposed "Modified Membrane Analogy" to express the elasto-plastic torsional stress distribution just before initial cracking. Analysis is based on this and contribution of steel to cracking strength is considered.

2.3.1 Modified Membrane Analogy

Fig. 2-1 shows the elasto-plastic condition of concrete just before initial cracking, plastic zone by sand-heap and elastic zone by membrane.

According to the analogical consept, torsional moment M_t corresponds to twice of the stress hill shown in Fig. 2-1. So, M_{tc} for the rectangular cross section is,

$$M_{tc}=\varphi_3 A_t b \tau_{t\max} \qquad (2.1)$$

Table 2-1

No.		reinforcement	α	M_{tu}/M_{tc}
A		longotidinal bars and stirrups	0	1.16
B				1.11
C				1.51
D				1.48
A	1 ~ 2	plain	0	1.00
D	1 ~ 2	lonhitudinal bars only	0	1.00
	3 ~ 4		1	1.44
	5 ~ 6		2	1.55
	7 ~ 8		3	2.15
F	1 ~ 2	longitudinal bars and stirrups	0	1.00
	3 ~ 4		1	1.91
	5 ~ 6		2	2.03
	7		3	2.17
	8		∞	3.60

Table 2-3

No.	α	K_1/K_2
A	0	0.043
B		0.034
C		0.163
D		0.066
A—1	0	0.095
〃—2		
B—1	0.65	0.067
〃—2	1.30	0.221
C—1	2.60	0.165
〃—2		

K_1 : torsional stiffness before oracking

K_2 : torsional stiffness after cracking

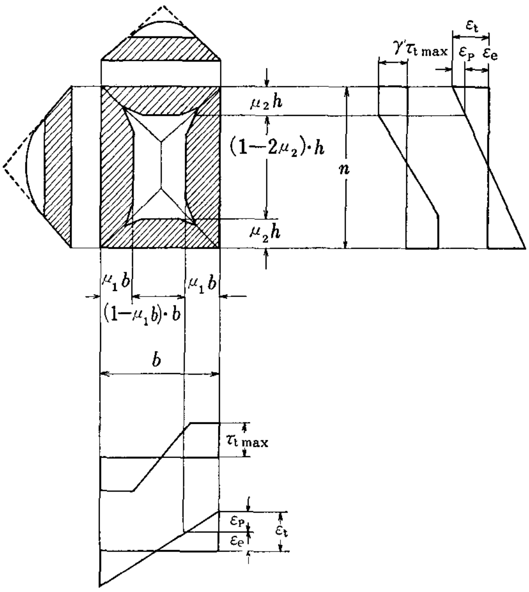


Fig. 2-1 elasto-plastic condition for torsion

$$\varphi_3=\frac{1}{2}-\frac{1}{6}\cdot\frac{b}{h}-(1-2\mu_1)^2$$
$$\times\left\{\left(\frac{1}{2}-\varphi_1\right)(1-\varphi_2)-\frac{1}{6}\cdot\frac{b}{h}(1-2\mu_1)\right\}$$

The coefficients φ_1 and φ_2 are obtained from experiments.

2.3.2 Torsional equivalent cross section

The member has the longitudinal bars and stirrups orthogonal to them, and they share the each direct component of the internal forces. In pure torsion, the components of torsional shear stress along to the longitudinal bar and stirrup are the same value $\tau_t/\sqrt{2}$. The effective area of stirrups is given as follows, (**Fig.** 2-2).

$$\frac{2a_{sv}h_1}{s}\cdot\frac{1}{\sqrt{2}}\lambda_1+\frac{2a_{sv}b_1}{s}\frac{1}{\sqrt{2}}q\lambda_2 \tag{2-2}$$

Then,

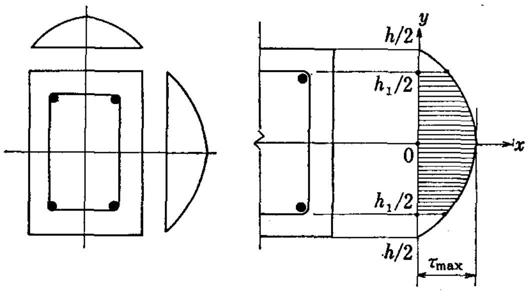


Fig. 2-2 effective area of stirrup

$$\left.\begin{aligned}&A_t=A_c+2\sqrt{2}\,n\frac{a_{sv}}{s}(h_1\lambda_1+b_1\lambda_2 q)\\&\lambda_1=1-\frac{1}{3}\left(\frac{h_1}{h}\right)^2,\quad \lambda_2=1-\frac{1}{3}\left(\frac{b_1}{b}\right)^2\end{aligned}\right\} \tag{2-3}$$

2.3.3. Torsional cracking moment M_{tc}

Using the equivalent cross sectional area and considering the elasto-plastic condition of torsional stress, M_{tc} is given as follows for the rectangular cross section,

$$\left.\begin{aligned}&M_{tc}=\varphi_3\{1+n(2\sqrt{2}u)\cdot r\}b^2h\cdot\tau_{t.max}\\&u=\frac{h_1}{h}\lambda_1+\frac{b}{h}\frac{b_1}{b}\lambda_2\cdot q\end{aligned}\right\} \tag{2-4}$$

2.3.4 Coefficients

The coefficients μ_1, μ_2, μ_3 and u which are used in the above expressions are as follows.

(1) φ_1, φ_2

Based on the experimental results of 32 plain concrete members μ_1 and μ_2 are given as the following expressions.

$$\left.\begin{aligned}&\mu_1=-0.02\left(\frac{h}{b}\right)^2+0.09\left(\frac{h}{b}\right)+0.13\\&\mu_2=-0.02\left(\frac{h}{b}\right)+0.09+0.13\left(\frac{b}{h}\right)\end{aligned}\right\} \tag{2-5}$$

Table 2-4

h/b	1	2	3	4	5
μ_1	0.210	0.230	0.230	0.175	0.075
μ_2	0.210	0.115	0.077	0.044	0.015
φ_3	0.309	0.373	0.306	0.386	0.341

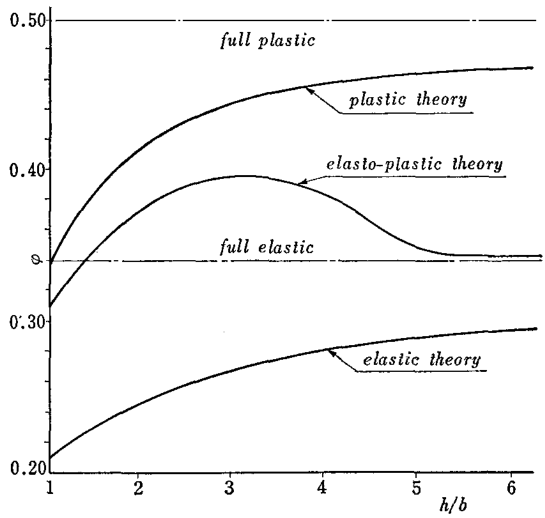
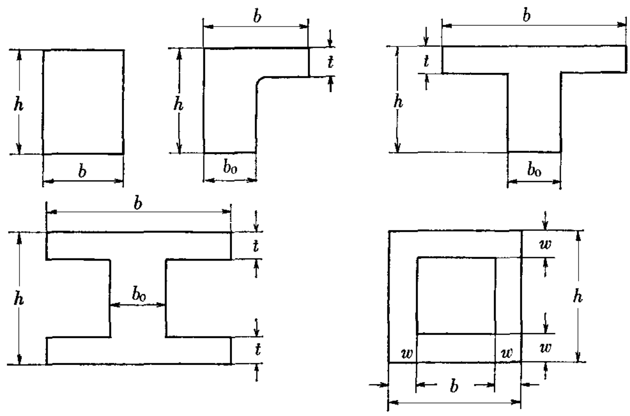


Fig. 2-3 torsional coefficients

Table 2-5

No.		dimention of cross section (cm)		num bers	σ_{tu} (kg/cm^2)	M_{tc} (kg·m)		
						test	calc.	test/calc.
R_-	1	$b=5$	$h=$ 5	5	35.0	16.3	13.5	1.21
	2		10	5		35.7	32.6	1.10
	3		15	5		58.0	52.0	1.12
	4		20	5		78.0	67.6	1.16
	5		25	5		100.5	74.6	1.34
L_-	1	$b_0=5$ $t=4$ $h=15$	$b=$ 7.5	6	35.0	71.6	54.6	1.31
	2		10.5	6		75.1	59.4	1.26
	3		12.5	6		79.3	65.6	1.21
	4		15.0	6		81.9	71.2	1.15
T_-	1		$b=$ 10	6	35.0	78.9	56.3	1.40
	2		15	6		87.9	66.4	1.31
	3		20	6		89.5	78.3	1.14
	4		25	6		94.4	85.4	1.11
I_-	1		$b=$ 10	5	35.0	92.0	57.3	1.61
	2		15	5		103.1	90.0	1.15
	3		20	5		126.2	107.8	1.17
	4		25	5		140.4	125.0	1.12



The concrete strength of these specimens is equal to 241~644 kg/cm^2 and the cross section ratio h/b is 1~4.

(2) φ_3

Torsional constant φ_3 for "Modified Membrane Analogy" can be calculated using the values μ_1 and μ_2 as shown in **Table 2-4**. The comparisons between φ_1, φ_2 and φ_3 are shown in **Fig. 2-3**.

(3) u

When d' is assumed to be 0.15d,

$$u=0.605\left(1+q\frac{b}{h}\right) \tag{2-6}$$

2.4 Experiments

2.4.1 Test-1

This is the pure torsion test of the plain mortar members with rectangular, L_-, T_-, I_- and hollow cross sections. The details of the specimens are shown in **Table 2-5**. About M_{tc}, the comparisons between the test and calculated values are shown in Table 2-5. Observation of the failure surface of the members shows that torsional failure is the same as the skew bending failure. This will be described later in the next chapter.

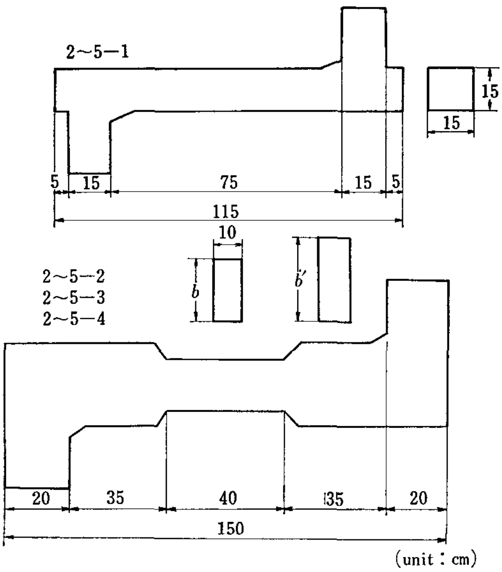
2.4.2 Test-2

This is the the pure torsion test of the plain concrete members with rectangular cross section. The details of the specimens are shown in **Table 2-7**. The values of μ_1 and μ_2 are obtained from

Table 2-7

σ_c : kg/cm²

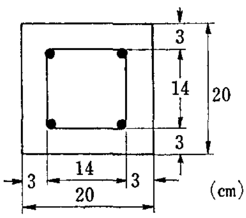
σ_c \ h/b	1	2	3	4
200	2—1	2—2	2—3	2—4
300	3—1	3—2	3—3	3—4
400	4—1	4—2	4—3	4—4
500	5—1	5—2	5—3	5—4



	b	b'
2～5—2	20	30
2～5—3	30	40
2～5—4	40	50

Table 2-8

A	1 2	0
B	1 2	0.65 1.30
C	1 2	2.60 2.60
D	1 2	∞ 0



longi. bars : 4D13
stirrups : ϕ9, s=10cm

these test results.

2.4.3 **Test-3**

The reinforced conrete members with the rectangular cross section were tested in pure torsion and combined bending and torsion. The details of the

Table 2-9-(1)

No.		α	σ_c(kg/cm²)	σ_{tu} (〃)
A	1 2	0	347	27.8
B	1 2	0.65 1.30	343	32.0
C	1 2	2.60 2.60	322	27.6
D	1 2	∞ 0	343	32.0

Table 2-9-(2) Margin of strength

No.		α	cr (tm)		sy (tm)		cu (tm)		syc/r	cu/cr
			M_b	M_t	M_b	M_t	M_b	M_t		
A	1	0	0.00	0.70	0.00	0.92	0.00	1.00	1.31	1.43
	2		0.00	0.70	0.00	0.95	0.00	1.00	1.36	1.43
B	1	0.65	0.29	0.45	0.60	0.95	0.67	1.00	2.10	2.22
	2	1.30	0.40	0.54	0.85	1.10	1.54	1.20	2.10	3.00
C	1	2.60	0.67	0.25	1.68	0.65	1.97	0.85	2.55	3.40
	2		0.54	0.20	2.03	0.78	2.12	0.85	3.83	4.25
D	1	∞	0.79	0.00	2.30	0.00	2.77	0.00	2.91	3.51
	2	0	0.79	0.00			0.00	0.90		

Table 2-9-(3) Margin of deformation

No.		α	cr		sy		cu		sy/cr	cu/cr
			θ	δ	θ	δ	θ	δ		
A	1	0	3.21	—	11.28	—	33.85	—	3.51	10.55
	2		3.87	—	25.34	—	63.90	—	6.55	16.51
B	1	0.65	0.03	34	17.14	227	26.01	310	6.68	9.12
	2	1.30	2.00	88	14.68	242	24.03	640	2.75	7.27
C	1	2.60	0.35	190	6.76	612	59.85	840	3.22	?
	2		0.79	98	10.24	700	26.86	850	7.14	?
D	1	∞	—	100	—	—	—	1900	—	—
	2	0								

(θ : 10^{-5} rad/cm, δ : 10^{-2}mm)

Table 2-9-(4) Crack width

No.		α	cr	sy	cu (just before)	sy/cr	cu/cr
A	1	0	0.05	0.25	0.60	5.00	12.00
	2		0.05	0.28	?	5.60	
B	1	0.65	0.05	0.25	?	5.00	
	2	1.30	0.05	0.30	0.75	6.00	25.00
C	1	2.60					
	2		0.05	0.25	0.75	5.00	25.00
D	1	∞	0.03	0.23	?	7.67	?
	2	0					

(mm)

Table 2-9-(5) Drop of stiffness

No.		α	K_1 0→cr	K_2 cr→sy	K_3 sy→cu	K_2/K_1	K_3/K_1
A	1	0	21.81	2.73	0.35	0.125	0.016
	2		18.09	1.16	0.13	0.064	0.007
B	1	0.65	43.69	2.92	—	0.067	—
	2	1.30	20.00	4.42	1.05	0.221	0.053
C	1	2.60	71.43	6.24	0.38	0.087	0.005
	2		25.32	6.13	0.42	2.242	0.017

(K : 10^8kg·cm^2)

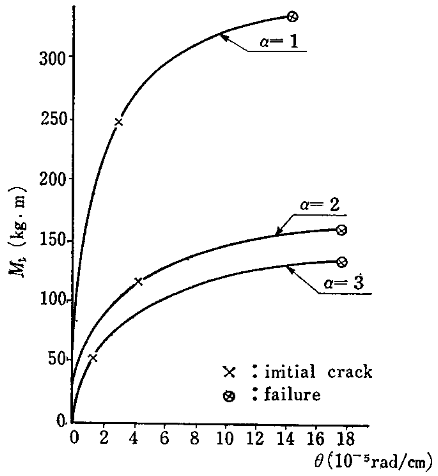


Fig. 2-4-(1)

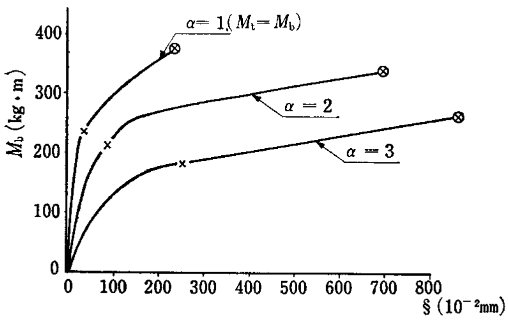
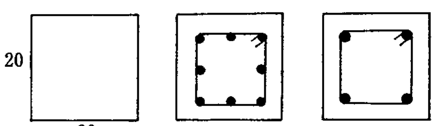


Fig. 2-4-(2)

Table 2-10 Test-5

		α
A	1 2	0.00
B	1 2	0.65 1.30
C	1 2	2.60
D	1 2	



A : plain

B : iongi. : 8ϕ9 stirruo : ϕ9 s = 7.5 cm

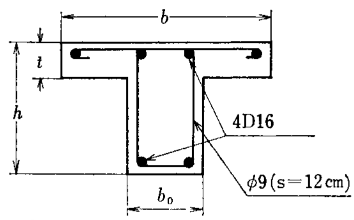
C, D : longi. : 4D13 stirrup : ϕ9 s = 7.5 cm

Table 2-11

			concrete (kg/cm^2)			M_{tc} (kg·m)		
			σ_{tu}	σ_c	E_c (10^5)	test	calc.	test / calc.
A	1	plain concrete	33.7	393	2.93	790	822	0.96
	2					790		0.96
B	1	reinforoed concrete	30.4	370	2.91	790	820	0.96
	2					833		1.01
B′	1					790		0.96
	2					830		0.01
C	1					830		1.04
	2					850		1.04
C′	1					—		—
	2					800		0.98

Table 2-12
Test-6

			b(cm)
A	1 2 3	RC	48
B	1 2 3		36
C	1 2 3		24
D	1 2 3		12
E	1 2 3	plain conc.	12



h = 30 cm
t = 8 〃
b_0 = 12 〃

A, B, C : T—section
D, E : rectangular section

specimens are shown in **Table** 2-8. The strength and deformation at initial cracking, crack width and drop of torsional stiffness just after cracking are shown in **Table** 2-9-(1)~(5)

2.4.4 Test-4

These are the pure torsion and combined bending and torsion tests of the plain and the reinforced concrete members with the rectangular cross section. The load-deformation curves (concerning torsion and bending respectively) are shown in **Fig.** 2-4-(1)~(2).

2.4.5 Test-5

This is the same test as Test-4 having the squre cross section. The details of the cross section are shown in **Table** 2-10 and the test results in

Table 2-11.

Test-6

This is the pure torsion test of the plain and reinforced concrete members with the rectangular and *T*- sections. The details of the specimens are shown in **Table** 2-12-(1)~(2). The margin of strength and deformation between initial cracking and failure and drop of torsional stiffness due to craking are shown in **Table** 2-13-(1)~(2). In **Table** 2-14 the compararisons of test and calculated values of M_{tc} are shown for all these tests.

2.5 Design method

2.5.1 Determination of cross section

First the conrete cross section is decided and then the reinforcing method is chosen. From Exp. (2-4),

$$h=\frac{M_{tc}}{b^2\sigma_{tu}}\times C,\quad C=\frac{1}{(1+2\sqrt{2}u\cdot n\cdot r)} \qquad (2\text{-}7)$$

The values of C are shown in **Table** 2-15, and these are aproximately equal to 2.600.

When M_{tc}, σ_{tu} and b are known or can be assumed, h may be calculated by Exp. (2-7), assuming that C is equal to 2.600. Using this values of h, C can be given by Exp. (2-7).

For reinforcement, r will be obtained by Table 2-15 corresponding to C and h/b. For the stirrup, a_{sv} and s ($s\leqq b_1$) can be decided from this r. For

Table 2-13-(1) Margth of strength and deformation

	strength			deformation		
	$M_{tc}(t\cdot m)$	M_{tu}(〃)	M_{tu}/M_{tc}	θ_c	θ_u	θ_u/θ_c
A	0.95	1.10	1.16	4.2	19.7	4.68
B	0.90	1.00	1.11	5.1	22.0	4.31
C	0.63	0.95	1.51	5.8	31.3	5.40
D	0.58	0.86	1.48	6.6	37.5	5.68
E	0.55	0.55	1.00	3.6	3.6	1.00

(θ : 10^{-5} rad/cm)

Table 2-13-(2) Drop of stffness

	K_1	K_2	K_2/K_1
A	22.54	0.97	0.043
B	17.63	0.59	0.034
C	10.79	1.23	0.163
D	8.52	0.76	0.086
E	14.44		

(K : 10^8 kg·cm²)

Table 2-14

Test No.	Specimen No.		M_{tc} (kg·m) test	calc.	test/calc.
2	R—	1	16.3	13.5	1.21
		2	35.7	32.6	1.10
		3	58.0	53.0	1.12
		4	78.5	67.6	1.16
		5	100.2	74.6	1.34
	L—	1	71.6	54.6	1.31
		2	75.1	59.4	1.26
		3	79.3	65.6	1.21
		4	81.9	71.2	1.15
	T—	1	78.9	56.3	1.40
		2	87.2	66.4	1.31
		3	89.5	78.3	1.14
		4	94.4	86.4	1.11
	I—	1	92.0	57.3	1.61
		2	103.1	90.0	1.15
		3	126.2	107.8	1.17
		4	140.4	125.0	1.12
3	A	1	700	734	0.95
		2	700	734	0.95
4	A	1 2	254	292	0.87
	D	1 2	310	292	1.06
	F	1 2	265	319	0.83
5	A	1	790	822	0.96
		2	790	822	0.96
6	D	1	600	609	0.99
		2	550	609	0.91
		3	550	609	0.91
	E	1	500	550	0.91
		2	550	550	1.00
		3	600	550	1.09

Table 2-15 C

r \ h/b	1.0	1.2	1.4	1.6	1.8	2.0
0.000	3.279	3.067	2.874	2.833	2.703	2.681
0.002	3.108	2.925	2.751	2.712	2.600	2.582
0.004	2.956	2.795	2.639	2.614	2.503	2.490
0.006	2.816	2.677	2.534	2.517	2.414	2.405
0.008	2.690	2.567	2.439	2.427	2.331	2.325
0.010	2.574	2.467	2.350	2.343	2.253	2.251
0.012	2.468	2.374	2.268	2.265	2.181	2.181
0.014	2.370	2.288	2.191	2.192	2.113	2.115
0.016	2.260	2.208	2.119	2.123	2.049	2.053
0.018	2.196	2.133	2.051	2.059	1.989	1.995
0.020	2.119	2.063	1.988	1.998	1.932	1.939

$$C=\frac{1}{1+\sqrt{2}\cdot 2\cdot unr} \quad (n=8)$$

the longitudinal bars,

$$As = \frac{a_{sv}(b_1 + h_1)}{s} \qquad (2\text{-}8)$$

2.5.2 Calculation of cracking strength

$$M_{tc} = C b^2 h \sigma_{tu} \qquad (2\text{-}9)$$

Using the values of C from Table 2-15 corresponding to h/b and r, M_{tc} can be calculated and safety for cracking will be checked.

3. Design method for ultimate strength[7)]

3.1 Summary

In this chapter a design method for the reinforced concrete members under combined bending and torsion is proposed from the view point of the ultimate strength design. This can be applied for the wide range of M_b/M_t ratio, from pure torsion to pure bending.

Before proposing the design method, failure mechanism of the member is analyzed and the general formulas are derived to estimate strength and deformation for yielding and failure conditons. This analysis is based on the "Skew Bending" concept and the calculating formulas are derived from the equilibrium and compatibility conditions.

The member which is considered here has the rectangular cross section and is reinforced with the logitudinal bars and stirrups orthogonal to them. With the increasing of load, the member cracks and the tension bars yield and finally fails by crushing of the compressive concrete.

The stress-strain relations for concrete and steel are decided to express accurately especially near the ultimate states. Based on the unique failure surface of the membre under torsion, the hypothetical member is set up and M_t and M_b acting to the actual member are transported into M_B and M_T to this hypothetical member. From the general expressions for strength and deformation of the hypothetical member, the design formulas can be obtained.

3.2 Notations

$\alpha = M_b/M_t$

$\beta = \sigma_{sy}/\sigma_{co}$

γ : strength reduction factor

δ : deflection

$\varphi_{1\sim3}$: torsional coefficient for elastic theory, plastic theory and elasto-plastic theory (Modified Membrane Analogy), respectively

ε_c : strain of concrete

ε_{co} : strain of concrete for it's maximum stress

ε_s : strain of steel

ε_{s1} : strain of longitudinal bar

ε_{s2} : strain of stirrup

ε_{sy} : strain of steel at yielding

ε_{sh} : strain of steel at the begining of the strain hardening

λ : corrective coefficient due to inclination of the neutral axis of the hypothetical member

σ_{co}, σ_{cu} : maximum and crushing stress of concrete, respectively

σ_{tu} : splitting tensile strngth of concrete

φ', e' : curvature and twist angle per unit length of the hypothetical member

φ, e : ,, of the actual member

ϕ, θ : rotation angle of the cross section and twisting angle of the actual member

A_s, A_s' : area of the longitudinal bars at the tension and compression side, respectively

a_{sv} : area of one leg of stirrup

b, h : length of the shorter and longer side of the cross section, respectively

d : effective depth

d' : cover thickness

E_c, E_s : young's modulus of concrete and steel, respectively

G : shear modulus of concrete

M_t, M_b : torsional and bending moment, respectively

l_z : length of torsional plsatic hinge

$n = E_s/E_c$

$p = A_s/bd, \quad p' = A_s'/bd, \quad r = a_{sv}/bs$

s : spacing of stirrups

$K = M_t/\theta$: torsional stiffness of the member, K_1 is for before crackinng and K_2 for after cracking

3.3 Stuctural analysis

3.3.1 Hypothetical member

The failure surface of the member under combined bending and torsion can be idealized as shown in **Fig.** 3-1.

The initial cracking changes the distribution of the internal forces into the condition of skew bending. The angle θ_1 is determined by the direction of the principal stress which is due to the torsional shear stress and bending tensile stress, and θ_3 is decided by θ_1, θ_2 and so on. The analysis of the failure mechanism needs the values of θ_1 and θ_3, and these values are shown in **Fig.** 3-3 which are obtained from the calculating and testing results.

M_t and M_b which act to the actual member are translated into the moments which act to the hypothetical member. In **Fig.** 3-4, M_t and M_b are shown as vectors in b), and these components about the failure surface are shown in c), and then the final condition is obtained as d). Then,

$$\left.\begin{aligned} M_B &= M_t\ (\cos\theta_3 + \alpha \sin\theta_3) \\ M_T &= M_t\ (\sin\theta_3 - \alpha \cos\theta_3) \end{aligned}\right\} \qquad (3\text{-}1)$$

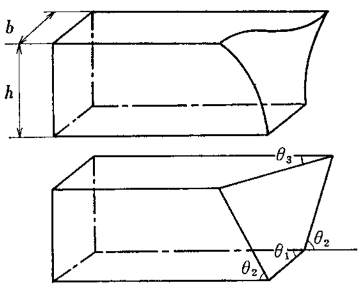


Fig. 3-1 faiure surface

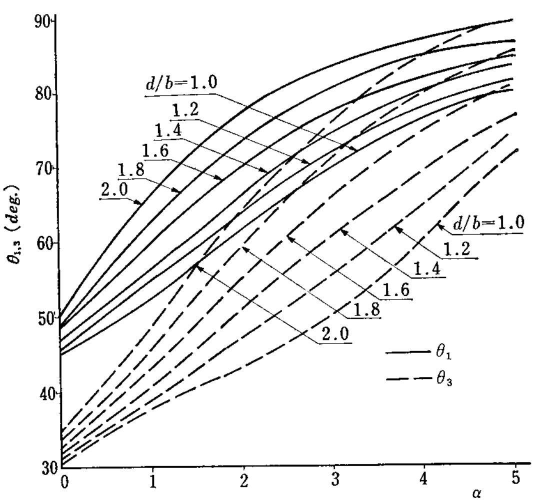


Fig. 3-3 θ_1 and θ_3

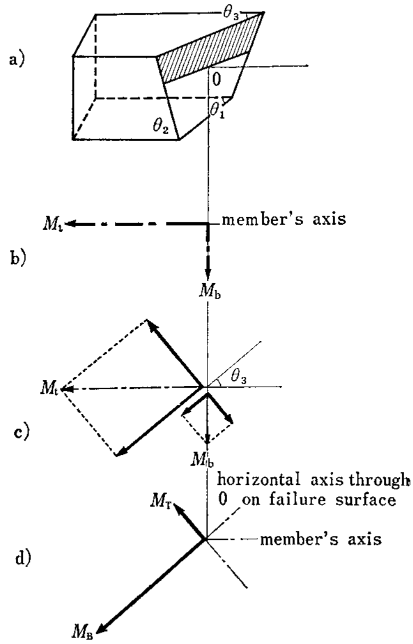


Fig. 3-4 M_t and $M_b \longrightarrow M_B$ and M_T

Based on the indealized failure surface, the hypothetical member is set up which has the inverted trapezoidal cross section and is double reinforced. Strength and deformation are obtained first about this hypothetical member and these are translated into the ones of the actual member. Using Exp. (3-1), M_B/M_t, M_T/M_t and M_B/M_T are calculated as shown in **Table** 3-1 for $\alpha = 0 \sim \infty$ and $h/b = 1.0 \sim 2.0$.

3.3.2 Expressions of stress-strain relationships for the materials

To analyze the strength and deformation of the member especially near the failure condition, the relationships between stress and strain of the concrete and steel must be decided.

(1) concrete :

$$\sigma_C = \left(\frac{\varepsilon_C}{\varepsilon_{CO}} e^{1-\frac{\varepsilon_c}{\varepsilon_{co}}}\right)\sigma_{CO} \qquad (3.2)$$

This formula shows grapphically the relation between σ_C/σ_{CO} and $\varepsilon_C/\varepsilon_{CO}$ in **Fig.** 3-5.

(2) steel :

$$\left.\begin{array}{ll} \varepsilon_S \leqq \varepsilon_{sy} & \sigma_S = E_S \varepsilon_S \\ \varepsilon_{sh} < \varepsilon_S \leqq \varepsilon_{sh} & \sigma_S = \sigma_{sy} \\ \varepsilon_{sh} < \varepsilon_S < \varepsilon_{S.\max} & \sigma_S = \left(1 + A \log_e \dfrac{\varepsilon_S}{\varepsilon_{sh}}\right) \cdot \sigma_{sy} \end{array}\right\} \qquad (3.3)$$

Table 3-1

h/b	α	M_B/M_t	M_T/M_t	M_B/M_T	h/b	α	M_B/M_t	M_T/M_t	M_B/M_T
	0	0.866	0.500	1.732		0	0.833	0.553	1.506
	0.2	0.954	0.360	2.650		0.2	0.938	0.400	2.345
	0.4	1.053	0.228	4.618		0.4	1.037	0.291	3.564
	0.65	1.192	0.020	59.60		0.65	1.187	0.128	9.266
1.0	1.0	1.404	−0.172	− 8.163	1.6	1.0	1.413	−0.054	−26.167
	1.5	1.730	−0.506	− 3.419		1.5	1.786	−0.248	− 7.202
	2.0	2.095	−0.781	− 2.682		2.0	2.211	−0.336	− 6.580
	3.0	2.941	−1.163	− 2.529		3.0	3.102	−0.612	− 5.069
	5.0	5.053	−0.683	− 7.398		5.0	5.099	−0.081	−62.051
	∞	∞	0.000	∞		∞	∞	0.000	∞
	0	0.857	0.515	1.664		0	0.824	0.566	1.456
	0.2	0.951	0.369	2.577		0.2	0.917	0.462	1.985
	0.4	1.053	0.228	4.618		0.4	1.029	0.138	3.236
	0.65	1.191	0.062	19.210		0.65	1.181	0.166	7.114
1.2	1.0	1.406	−0.148	− 9.500	1.8	1.0	1.414	0.000	
	1.5	1.751	−0.431	− 4.063		1.5	1.780	−0.104	−17.116
	2.0	2.145	−0.633	− 3.389		2.0	2.228	−0.192	−11.604
	3.0	3.031	−0.090	− 3.360		3.0	3.162	−0.004	−790.50
	5.0	5.032	−0.417	−12.190		5.0	5.077	0.472	10.756
	∞	∞	0.000	∞		∞	∞	0.000	∞
	0	0.848	0.530	1.600		0	0.817	0.576	1.418
	0.2	0.949	0.377	2.517		0.2	0.866	0.445	1.946
	0.4	1.049	0.246	4.264		0.4	1.023	0.336	3.045
	0.65	1.190	0.083	15.340		0.65	1.175	0.202	5.817
1.4	1.0	1.409	−0.123	−11.460	2.0	1.0	1.412	0.074	19.081
	1.5	1.768	−0.354	− 4.994		1.5	1.803	0.003	601.000
	2.0	2.179	−0.501	− 4.349		2.0	2.236	−0.009	−248.44
	3.0	3.098	−0.634	− 4.886		3.0	3.155	0.223	14.148
	5.0	5.093	−0.239	− 2.131		5.0	5.032	0.825	6.099
	∞	∞	0.000	∞		∞	∞	0.000	∞

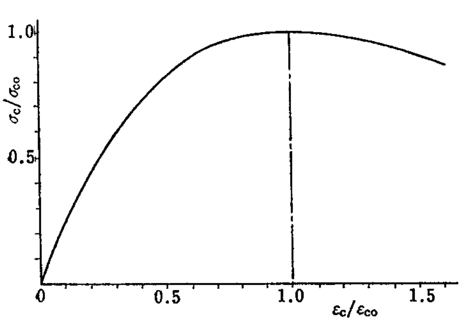


Fig. 3-5 for concrete

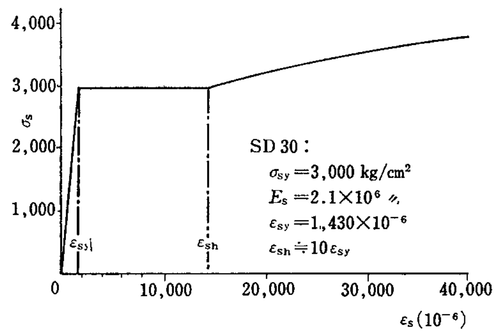


Fig. 3-6 for steel

The $\sigma_s - \varepsilon_s$ curve for the following condition is shown as **Fig. 3-6**.

$\sigma_{sy} = 3000\,\mathrm{kg/cm^2}$, $E_s = 2.1 \times 10^6\,\mathrm{kg/cm^2}$

$A = 0.25$, $\varepsilon_{sh} = 10\,\varepsilon_{sy}$

3.3.3 General formulas for strength

The resisting moment of the hypothetical member is obtained provided that the equilibrium and compatibility conditions are both satisfied, and then the general formulas for strength of the actual member is introduced based on this.

The hypothetical cross section under M_B and M_T is reinforced by the longitudinal bars and stirrups in the tension and compression sides. The relation between the directions of these steels and the inter-

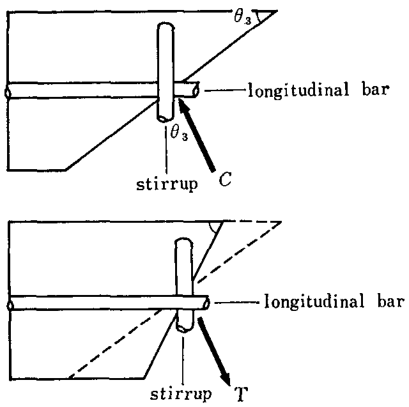


Fig. 3-7 direction of steel and internal forces

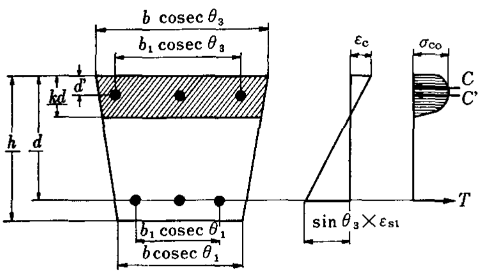


Fig. 3-8 hypothetical cross section

nal forces is shown in **Fig.** 3-7. As these directions do not agree with each other, the correct by the angle θ_3 is needful for the equilibrium and compatibility conditions.

(1) Coefficients of the neutral axis

In the condition shown as **Fig.** 3-8,

$$C=bd\sigma_{co}\left[\operatorname{cosec}\theta_3 e\left\{\frac{1}{\eta}-e^{-\eta}\left(1+\frac{1}{\eta}\right)\right\}\cdot K-\frac{a}{\eta}\right.$$

$$\times(\operatorname{cosec}\theta_3-\operatorname{cosec}\theta_1)$$

$$\left.\times\frac{e}{\eta}\left\{(1+e^{-\eta})-\frac{2}{\eta}(1-e^{-\eta})\right\}K^2\right]$$

$$C'=\sin\theta_3 A_s'\sigma_s'_{1B}+a_{sv}'\frac{1.5b\cot\theta_3}{s}\cos\theta_3\cdot\sigma_s'_{2B}$$

$$=bd\sigma_{co}\left[\sin\theta_3 p'\frac{\sigma_s'_{1B}}{\sigma_{co}}+1.5\frac{b}{d}\cot\theta_3\cos\theta_3 r'\frac{\sigma_s'_{2B}}{\sigma_{co}}\right]$$

$$T=bd\sigma_{co}\left[\sin\theta_3\cdot p\frac{\sigma_{s1B}}{\sigma_{co}}+1.5\frac{b}{d}\cot\theta_1\cdot\cos\theta_3 r\frac{\sigma_{s2B}}{\sigma_{co}}\right]$$

$$C+C'-T=0$$

$$C_1\eta_1 k-C_2\eta_2 k^2+C_3 p'\frac{\sigma_s'_{1B}}{\sigma_{co}}-C_4 p\frac{\sigma_{s1B}}{\sigma_{co}}=0 \qquad (3\text{-}4)$$

$$C_2=\frac{d}{h}(\operatorname{cosec}\theta_3-\operatorname{cosec}\theta_1),$$

$$C_3=\sin\theta_3+\cot^2\theta_3\cos\theta_3$$

$$C_4=\sin\theta_3+\cot^2\theta_1\cos\theta_3$$

$$\eta_1=e\left\{\frac{1}{\eta}-e^{-\eta}\left(1+\frac{1}{\eta}\right)\right\},$$

$$\eta_2=\frac{e}{\eta}\left\{(1+e^{-\eta})-\frac{2}{\eta}(1-e^{-\eta})\right\}$$

$$\eta=\frac{\varepsilon_c}{\varepsilon_{co}}=\sin\theta_3\frac{k}{1-k}\cdot\frac{\varepsilon_{sy}}{\varepsilon_{co}}$$

(2) Coefficients of the resisting moment

For the hypothetical member,

$$\frac{M_B}{bd^2\sigma_{co}}=C_1\eta_3 k^2+C_2\eta_4 k^3+C_3 p'\frac{\sigma_s'_{1B}}{\sigma_{co}}\left(K-\frac{d'}{d}\right)$$

$$+Cp\frac{\sigma_{s1B}}{\sigma_{co}}(1-k) \qquad (3\text{-}5)$$

Then for the actual member,

$$\frac{M_t}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\cdot\frac{M_B}{bd^2\sigma_{co}}$$

$$=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3 bd^2\sigma_{co}}\left\{C_1\eta_3 k^2+C_2\eta_4 k^3\right.$$

$$\left.+C_3 p'\frac{\sigma_{s1B}}{\sigma_{co}}\left(k-\frac{d'}{d}\right)+C_4 p\frac{\sigma_{s1B}}{\sigma_{co}}(1-k)\right\}$$

$$(3\text{-}6)$$

$$\eta_3=-e^{1-\eta}\left(1+\frac{2}{\eta}+\frac{2}{\eta^2}\right)+e\left(\frac{2}{\eta^2}\right)$$

$$\eta_4=-e^{1-\eta}\left(\frac{1}{\eta}+\frac{4}{\eta^2}+\frac{6}{\eta^3}\right)+e\left(-\frac{2}{\eta^2}+\frac{6}{\eta^3}\right)$$

σ_{s1B}, σ_{s2B} : stress of longitudinal bars of tensile and compressive side due to M_B, respecctively

3.3.4 General formulas for deformation

Deformation of the reinforced concrete members under combined bending and torsion is consists of twisting angle about the member's axis and rotation angle of the cross section about the axis orthogonal to the member's axis.

(1) At yielding

The twisting angle per unit length of the hypothetical member $\theta'_{t.sy}$ and curvature φ'_{sy} are expressed as follows from the condition of **Fig.** 3-9.

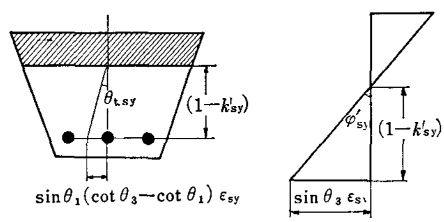


Fig. 3-9

$$\left.\begin{aligned}\theta'_{t.sy}&=\sin\theta_1(\cot\theta_3-\cot\theta_1)\frac{1}{d}\cdot\frac{\varepsilon_{sy}}{1-k_{sy}}\\ \varphi'_{sy}&=\sin\theta_3\frac{1}{d}\cdot\frac{\varepsilon_{sy}}{1-k_{sy}}\end{aligned}\right\}\quad(3\text{-}7)$$

The component of $\theta'_{t.sy}$ and φ'_{sy} along the direction of the member's axis is the twisting angle of actual member $\theta_{t.sy}$, and the component along the direction ohthogonal to the member's axis is the rotation angle of the actual member, The torsional plastic hinge is formed with yielding and its length is $l_z=b\cdot\cot\,\theta_3$, so the twisting angle of the member θ_{sy} and rotation angle ϕ_{sy} are,

$$\left.\begin{aligned}\theta_{sy}&=\theta_{t.sy}l_z=C_8\frac{\varepsilon_{sy}}{1-k_{sy}}\\ \phi_{sy}&=\varphi_{sy}l_z=C_9\frac{\varepsilon_{sy}}{1-k_{sy}}\end{aligned}\right\}\quad(3\text{-}8)$$

$$C_8=\frac{b}{d}\cos\theta_3(\cos\theta_3+\sin\theta_1\cot\theta_3-\cos\theta_1)$$

$$C_9=\frac{b}{d}\cot\theta_3\{\sin^2\theta_3+\sin\theta_1\cos\theta_3(\cot\theta_3-\cot\theta_1)\}$$

(2) At failure

For the hypothetical member, following the same consideration as at failure,

$$\left.\begin{aligned}\theta'_{t.cu}&=\sin\theta_1(\cot\theta_3-\cot\theta_1)+\cos\theta_3\frac{1}{d}\cdot\frac{\varepsilon_{cu}}{k_{cu}}\\ \varphi'_{cu}&=\frac{1}{d}\cdot\frac{\varepsilon_{cu}}{k_{cu}}\end{aligned}\right\}\quad(3\text{-}9)$$

For the actual member,

$$\left.\begin{aligned}\theta_{t.cu}&=\{\sin\theta_1(\cot\theta_3-\cot\theta_1)+\cos\theta_3\}\frac{1}{d}\cdot\frac{\varepsilon_{cu}}{k_{cu}}\\ \varphi_{cu}&=\{\cot\theta_3\sin\theta_1(\cot\theta_3-\cot\theta_1)+\sin\theta_3\}\frac{1}{d}\cdot\frac{\varepsilon_{cu}}{k_{cu}}\end{aligned}\right\}\quad(3\text{-}10)$$

The length of the plastic hinges is $lz=b\cdot\cot\theta_3$, so the twisting and rotation angles are,

$$\left.\begin{aligned}\theta_{cu}&=\theta_{t.cu}l_z=C_{10}\frac{\varepsilon_{cu}}{k_{cu}}\\ \phi_{cu}&=\varphi_{cu}l_z=C_{11}\frac{\varepsilon_{cu}}{k_{cu}}\end{aligned}\right\}\quad(3\text{-}11)$$

$$C_{10}=\cot\theta_3\{\cos\theta_3+\sin\theta_1(\cot\theta_3-\cot\theta_1)\}\frac{b}{d}$$

$$C_{11}=\cot\theta_3\{\sin\theta_3+\cot\theta_3\sin\theta_1(\cot\theta_3-\cot\theta_1)\}\frac{b}{d}$$

3.3.5 Strength and deformation at yielding of the member

When the memoer is reinforced within the balanced steel ratio, the member cracks with increasing of the load, and the tension bars yield, and then crushing of the compressive concrete leads the member to failure. The torsional plastic hinge forms with yielding and terminates with crushing.

At yielding, from the condition that strain of tensile steel becomes its yielding value,

$$\sigma_s=\sigma_{sy}$$

$$\varepsilon_s'=\frac{k=\frac{d'}{d}}{1-k}\varepsilon_{sy}\leqq\varepsilon'_{sy}\longrightarrow\sigma'_s=E_s\sigma_s'=\sigma_{sy}\frac{k-\frac{d'}{d}}{1-k}$$

$$\varepsilon_c=\sin\theta_3\,\varepsilon_{syt}\frac{k}{1-k}\longrightarrow\eta=\frac{\varepsilon_c}{\varepsilon_{co}}=\sin\theta_3\frac{\varepsilon_{sy}}{\varepsilon_{co}}\cdot\frac{k}{1-k}$$

Putting these expressions in Exp. (3-4) and (3-5),

$$C_1\eta_1k-C_2\eta_2k^2+C_3p'\beta'\frac{k-\frac{d}{d}}{1-k}-C_4p\beta=0\quad(3\text{-}12)$$

$$\frac{M_t}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\left\{C_1\eta_3k^2+C_2\eta_4k^3+C_3p'\beta'\frac{\left(k-\frac{d'}{d}\right)^2}{1-k}+C_4p\beta(1+k)\right\}\quad(3\text{-}13)$$

Deformation has been obtained before,

$$\theta_{sy}=C_8\frac{\varepsilon_{sy}}{1-k_{sy}},\qquad\phi_{sy}=C_9\frac{\varepsilon_{sy}}{1-k_{sy}}\quad(3\text{-}8)$$

3.3.6 Strength and deformation at failure of the member

From the condition that strain of the compressive concrete reaches to its ultimate values,

$$\varepsilon_s'=\cos ec\theta_3\,\varepsilon_{cu}\frac{k-\frac{d'}{d}}{k},\qquad\varepsilon_s=\cos ec\theta_3\,\varepsilon_{cu}\frac{1-k}{k},\qquad n=\frac{\varepsilon_{cu}}{\varepsilon_{co}}$$

(1) $\varepsilon_{sh}>\varepsilon_s'$, $\quad\varepsilon'_{sy}<\varepsilon_s'$

$$C_1\eta_1 k-C_2\eta_2 k^2+C_3 p'\beta'-C_4\left(1+A\log_e\frac{\varepsilon_s}{\varepsilon_{sh}}\right)p\beta=0 \tag{3-14}$$

$$\frac{M_t}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\left\{C_1\eta_3 k^2+C_2\eta_4 k^3+C_3 p'\beta'\left(k-\frac{d'}{d}\right)+C_4\left(1+A\log_e\frac{\varepsilon_s}{\varepsilon_{sh}}\right)p\beta(1-k)\right\} \tag{3-15}$$

(2) $\varepsilon_{sh}<\varepsilon_s$, $\varepsilon_s'<\varepsilon'_{sy}$

$$C_1\eta_1 k-C_2\eta_2 k^2+C_3 p'\frac{E_s\varepsilon_{cu}}{C_{12}\sigma_{co}}\cdot\frac{k-\frac{d'}{d}}{k}-C_4\left(1+A\log_e\frac{\varepsilon_s}{\varepsilon_{sh}}\right)p\beta=0 \tag{3-16}$$

$$\frac{M_t}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\left\{C_1\eta_3 k+C_2\eta_4 k^3+C_3 p'\frac{E_s\varepsilon_{cu}}{C_{12}\sigma_{co}}\cdot\frac{\left(k-\frac{d'}{d}\right)^2}{k}+C_4\left(1+A\log_e\frac{\varepsilon_s}{\varepsilon_{sh}}\right)p\beta(1-k)\right\} \tag{3-17}$$

(3) $\varepsilon_{sy}\leqq\varepsilon_s<\varepsilon_{sh}$, $\varepsilon_s'\leqq\varepsilon'_{sy}$

$$C_1\eta_1 k-C_2\eta_2 k^2+C_3 p'\frac{E_s\varepsilon_{cu}}{C_{12}\sigma_{co}}\cdot\frac{k-\frac{d'}{d}}{k}-C_4 p\beta=0 \tag{3-18}$$

$$\frac{M_t}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\left\{C_1\eta_3 k^2+C_2\eta_4 k^3+C_3 p'\frac{E_s\varepsilon_{cu}}{C_{12}\sigma_{co}}\cdot\frac{\left(k-\frac{d'}{d}\right)^2}{k}+C_4 p\beta(1-k)\right\} \tag{3-19}$$

(4) $\varepsilon_{sy}\leqq\varepsilon_s<\varepsilon_{sh}$, $\varepsilon'_{sy}<\varepsilon_s'$

$$C_1\eta_1 k-C_2\eta_2 k^2+C_3 p'\beta'-C_4 p\beta=0 \tag{3-20}$$

$$\frac{M_t}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\left\{C_1\eta_3 k^2+C_2\eta_4 k^3+C_3 p'\beta'\left(k-\frac{d'}{d}\right)+C_4 p\beta(1-k)\right\} \tag{3-21}$$

Deformation is,

$$\theta_{cu}=C_{10}\frac{\varepsilon_{cu}}{k_{cu}},\qquad \phi_{cu}=C_{11}\frac{\varepsilon_{cu}}{k_{cu}} \tag{3-11}$$

3.3.7 Calculations using above expressions

(1) Stress due to M_T

Tensile steel stress due to M_T is,

longitudinal bar:

$$\sigma_{S1T}=K_1\frac{1}{p(1-k)}\cdot\frac{1}{bd^2}\times M_t,\quad K_1=\frac{\sin\theta_3-\alpha\cos\theta_3}{16\cos\theta_1}$$

stirrup:

$$\sigma_{S2T}=-K_2\frac{d}{1.5b}\cdot\frac{p}{r}\sigma_{S1T},\quad K_2=\cot\theta_3\cdot\tan\theta_1$$

Calculating resuts show that the ratio $\sigma_{s1T}/\sigma_{s1y}$ and $\sigma_{s2T}/\sigma_{s2y}$ are generally within 5%, so M_T may be neglected.

(2) Yield strength

The neutral axis and torsional moment coefficients are expressed as Exp. (3-12) and (3-13). In these expressions $\eta_{1\sim4}$ can be calculated for various values of d/b, α and k. λ is the correct coefficient about inclination of the neutral axis. Assuming that $\varepsilon_{sy}=\frac{3000}{2.1\times10^6}=1430\times10^{-6}$(SD 30) and $\varepsilon_{co}=3000\times10^{-6}$, the calculating results of k_{sy} and $M_{t.sy}/bd^2\sigma_{co}$ are shown in Fig. 3-21 and Fig. 3-15.

(3) Ultimate strength

The ultimate strength is expressed as Exp. (3-14)～(3-21). Assuming that $\varepsilon_{co}=3000\times10^{-6}$ and $\varepsilon_{cu}=4500\times10^{-6}$, $\eta_{1\sim4}$ becomes constant.

When $\varepsilon_{sh}<\varepsilon_s$, $\varepsilon'_{sy}<\varepsilon_s'$,

$$0.8013\,C_1 k_{cu}-0.3394\,C_2 k^2_{cu}+C_3\left(1+0.5757\,\mathrm{lgo}_e\frac{\varepsilon_s}{\varepsilon_{sh}}\right)p\beta=0 \tag{3.22}$$

$$\frac{M_{t.cu}}{bd^2\sigma_{co}}=\frac{\lambda}{\cos\theta_3+\alpha\sin\theta_3}\left\{0.4619\,C_1 k^2_{cu}-0.3394\,C_2 k^2_{cu}+C_3\left(k_{cu}-\frac{d'}{d}\right)p'\beta'+C_4\left(1+0.5757\log_{10}\frac{\varepsilon_s}{\varepsilon_{sh}}\right)(1-k)p\beta\right. \tag{3.23}$$

Generally k_{cu} is small at ultimate condition, so calculation can be neglected for the case of (2)～(4). Assuming that $\varepsilon_{sh}=10\,\varepsilon_{sy}=14300\times10^{-6}$, k_{cu} is shown graphically in **Fig.** 3-10 and $M_{t.cu}/bd^2\sigma_{co}$ in **Fig.** 3-11.

(4) Deformation

Deformation at yielding and ultimate conditions are expressed in Exp. (3-8) and (3-11). In these formulas $C_{8\sim11}$ are the functions of θ_1, θ_3 and b/h and these values are calculated in **Fig.** 3-12 for the various combinatins of α and b/d.

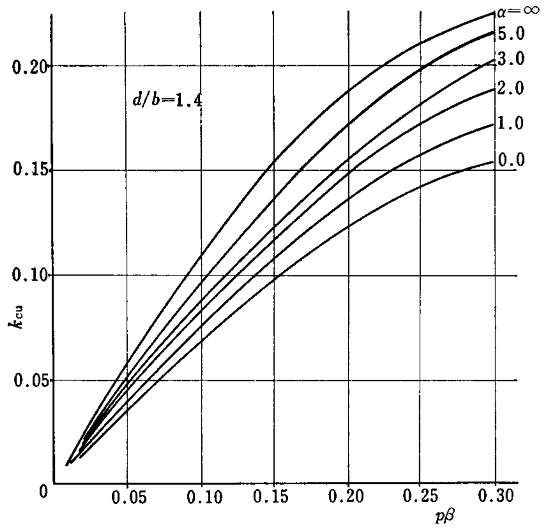


Fig. 3-10 k_{cu}

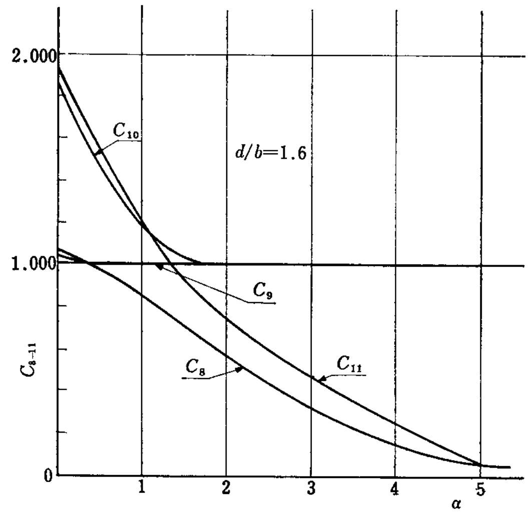


Fig. 3-12 $C_{8\sim 11}$

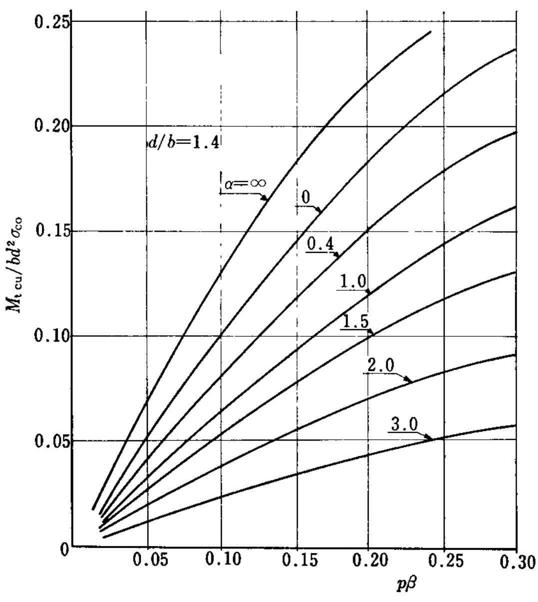


Fig. 3-11 $p\beta$——$M_{t.cu}/bd^2\sigma_{CO}$

3.4 Reinforcing method

Generally in torsion, margin of strength from initial cracking to failure is small and drop of torsional stiffness with cracking is remarkable. One of the purposes of reinforcing for torsion is to make this drop moderate.

For torsion it is effective to use the spiral reinforcement, but this is difficult both in design and in practice especially for combined bending and torsion. So the longitudinal bars and stirrups orthogonal to them are used, and in this case the former takes charge of longitudinal component of the internal forces and the latter orthogonal component. Adapting this method, it does not need to change the steel arrangement acoompanied with twisting direction. By suitable selection of the propotion for the longitudinal bars and stirrups ratio accrding to the bending and torsional moment ratio and to the cross section, the members can be reinforced effectively and economically.

3.4.1 Propotion of stirrup and longitudinal bar ratio

In Fig. 3-7, the direction of internal forces and the location of the steel are shown. T_1 is the component of tensile force T for the longitudinal bars' direction and T_2 for the stirrups'.

$$T_1 = T\sin\theta_3 = A_s\sigma_{s1}$$

$$T_2 = T\cos\theta_3 = a_{sv}\frac{1.5b\cot\theta_1}{s}\sigma_{s2}$$

From the condion that both steels yield at the same time,

$$\operatorname{cosec}\theta_3 A_s\sigma_{s1y} = \sec\theta_3 a_{sv}\frac{1.5b}{s}\sigma_{s2y}$$

Then,

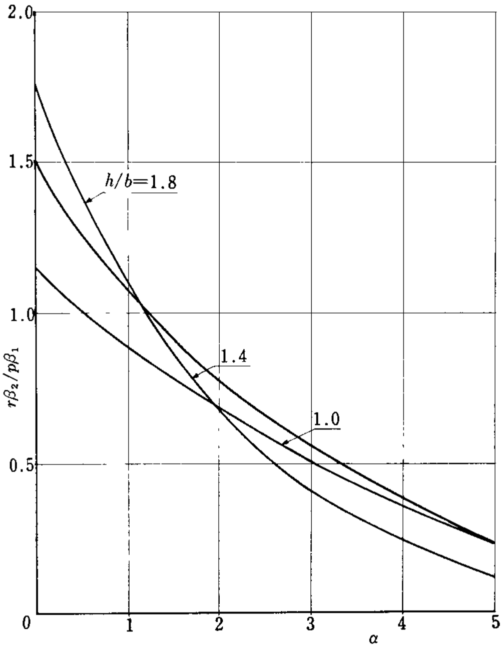


Fig. 3-14 $\alpha-r\beta_2/p\beta_1$

$$\frac{r\beta_2}{p\beta_1}=\frac{d}{b}\cdot\frac{1}{1.5}\cdot\cot\theta_3 \tag{3-24}$$

$\sigma_{s1}, \sigma_{s1y}$: stress and yield stress of the tensile longitudinal bar, respectively

ρ_{s2}, σ_{s2y} : the same of the stirrup

$\beta_1=\sigma_{s1y}/\sigma_{co}$, $\beta_2=\sigma_{s2y}/\sigma_{co}$

The relations between α and $r\beta_2/p\beta_1$ are calculated and shown in **Fig.** 3-14.

3.4.2 Longitudinal steel ratio

It is proper for design of torsion to consider the ultimate condition as yielding of the member. The yielding strength $M_{t.sy}$ is expressed as Exp. 3-13. Neglecting $C_2 n_4 k^3{}_{sy}$ and putting $p'\beta'$ equal to $0.5p\beta_1$ in this formula,

$$p\beta_1=\frac{\dfrac{M_{t.sy}}{bd^2\sigma_{co}}\cdot\dfrac{\cos\theta_3+\alpha\sin\theta_3}{\lambda}-C_1\eta_3 k^2{}_{sy}}{C_3\left\{\dfrac{1}{2}\cdot\dfrac{\left(k_{sy}-\dfrac{d'}{d}\right)^2}{1-k_{sy}}+(1-k_{sy})\right\}} \tag{3-25}$$

About k_{sy} in the above formula the following approximate expressions can be obtained from the calculating results using Exp. (3-12).

$$\left.\begin{array}{lr} 5.0\geqq\alpha>0 & \\ \quad k=0.193+1.610p\beta_1 & 0.125\geqq p\beta\geqq 0 \\ \quad k=0.310+0.680p\beta & p\beta>0.125 \\ \infty\geqq\alpha>5.0 & \\ \quad k=0.165+1.610p\beta_1 & 0.125\geqq p\beta\geqq 0 \\ \quad k=0.274+0.680p\beta_1 & p\beta>0.125 \end{array}\right\} \tag{3-26}$$

When assumed or known α and $M_{t.sy}$ about the load, b and d about the cross section, and σ_{co} and σ_{sy} about the materials are, longitudinal steel ratio $p\beta_1$ can be obtained by Exp. (3-25). The stirrup ratio $r\beta_2$ can be also calculated by Exp. (3-24).

According to the combinations of α and h/b, the relations between $p\beta_1$ and $M_{t.sy}/bd^2\sigma_{co}$ are calculated and shown in **Fig.** 3-15-(1)~(2), and for $M_{t.cu}/bd^2\sigma_{co}$ in **Fig.** 3-16-(1)~(2).

3.4.3 Location of the steel

The spacing of stirrups s may be decided as

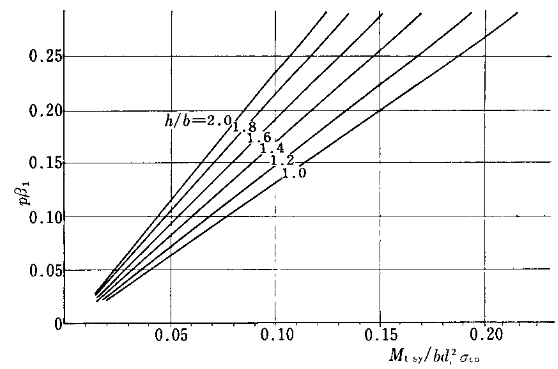


Fig. 3-15-(1) $p\beta_1-M_{t.sy}/bd\sigma_{cn}$ $(\alpha=0)$

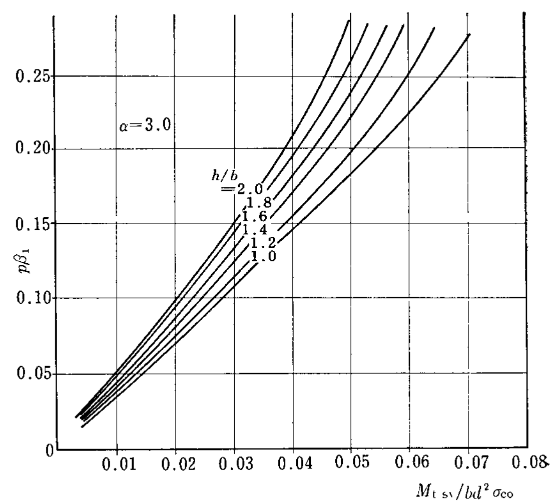


Fig. 3-15-(2) $p\beta_1-M_{t.sy}/bd^2\sigma_{co}$ $(\alpha=3.0)$

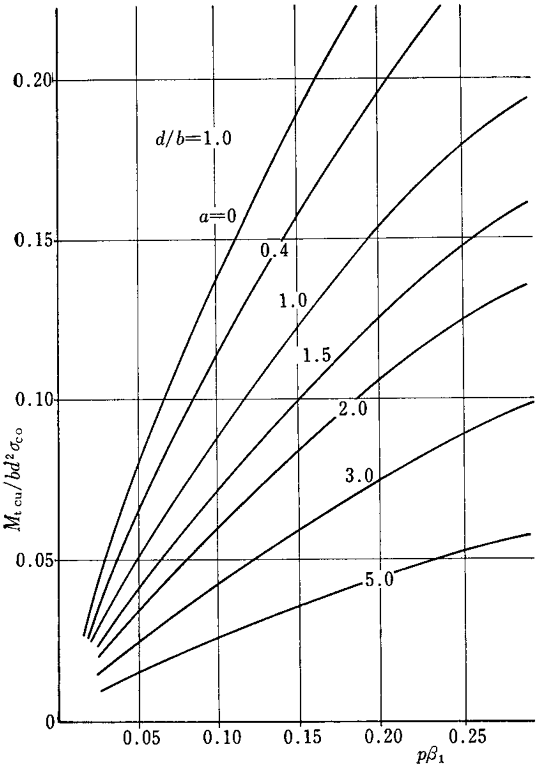


Fig. 3-16-(1) $p\beta_1-M_{t.cu}/bd^2\sigma_{co}$

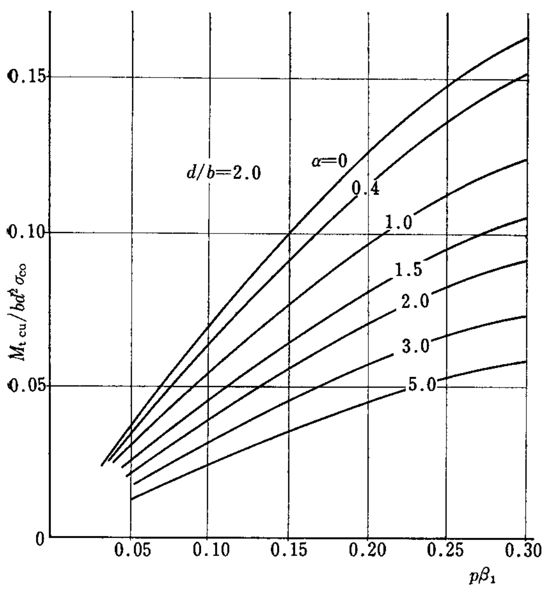


Fig. 3-16-(2) $p\beta_1-M_{t.cu}/bd^2\sigma_{co}$

follows for the values of r $(=a_{sv}/bs)$. Stress of the longitudinal bars σ_{s1} and of stirrups σ_{s2} are,

$$\sigma_{s1}=\frac{T\sin\theta_3}{A_s},\qquad \sigma_{s2}=\frac{T\cos\theta_3}{a_{sv}\frac{b}{s}\times 1.5}$$

From the condition that σ_{s1} does not exceed σ_{s2}, the spacing s is,

$$\left.\begin{aligned} &s\leqq(1.5\cdot\tan\theta_3)\frac{a_{sv}}{A_s}b \\ &s\leqq b,\quad s\leqq\frac{h}{2}(b\leqq h) \end{aligned}\right\}\qquad (3\text{-}27)$$

3.4.4 Steel ratio and deformable ability

Deformation from yielding to crushing is dependent on mainly the steel ratio, and the relations between the steel ratio and this deformable ability are shown in **Fig.** 3-13.

3.4.5 Balanced steel ratio

The balanced steel ratio can be given from the condition that the tensile steel's strain reaches its yiild strain when the concrete's strain becomes its compressive ultimate strain.

When ε_s is equal to ε_{sy} and ε_s' to $\dfrac{k-\frac{d'}{d}}{1-k}\varepsilon_{sy}$,

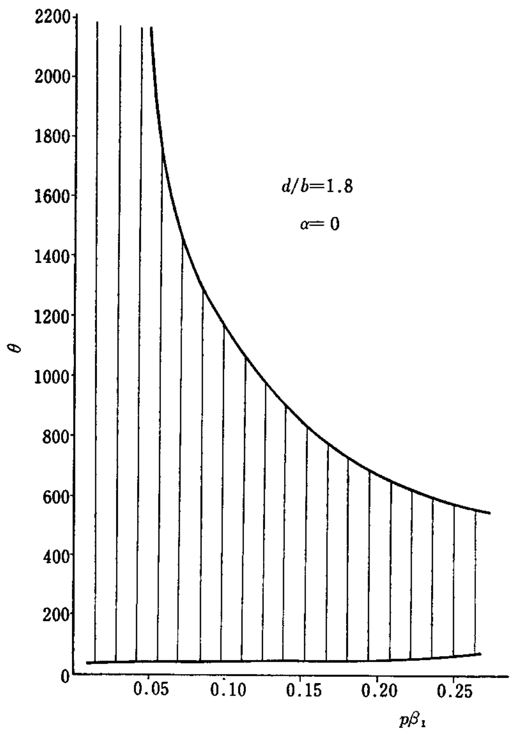


Fig. 3-13 $p\beta_1-\theta$

$$C_1\eta_1 k - C_3\eta_2 k^2 + C_3 p'\beta' \frac{k-\frac{d'}{d}}{1-k} - C_3 p_0\beta_0 = 0$$

From the strain condition,

$$k = \frac{\varepsilon_{cu}}{\varepsilon_{cv} + \sin\theta_3 \varepsilon_{sy}} = \frac{1}{1+\sin\theta_3 \zeta}$$

Based on these two formulas,

$$p_0\beta_0 = \frac{\frac{C_1}{C_3}\cdot\frac{\eta_1}{1+\zeta} - \frac{C_2}{C_3}\cdot\frac{\eta_2}{(1+\zeta)^2}}{1-\frac{p'\beta'}{p_0\beta_0}\left\{\frac{1}{\zeta}-\left(1+\frac{1}{\zeta}\right)\frac{d'}{d}\right\}} \qquad (3\text{-}26)$$

An example of $p_0\beta_0$ is shown in **Table** 3-4, assuming that ε_{cu} is equal to 4500×10^{-6} and ε_{sy} to 1430×10^{-6} (SD30).

3.4.6 Bending and torsional stiffness

Table 3-2 : $K_{1\sim4}$ ($h/b=1.4$)

α \ K \ pβ	K	0.05	0.10	0.15	0.20	0.25
0	1	19.09	32.48	43.77	54.13	65.03
	2	0.22	0.71	1.36	2.13	2.99
	3	—	—	—	—	—
	4	—	—	—	—	—
0.4	1	17.66	29.83	40.26	51.48	59.62
	2	0.24	0.74	1.42	2.22	3.11
	3	7.42	12.45	16.75	20.51	24.70
	4	0.09	0.30	0.59	0.92	1.29
1.0	1	—	—	—	—	—
	2	0.27	0.86	1.56	2.54	3.57
	3	—	—	—	—	—
	4	0.28	0.88	1.61	2.62	3.68
1.5	1	18.75	31.61	39.73	45.00	51.72
	2	0.31	0.99	1.37	2.90	4.07
	3	22.50	38.79	47.18	54.51	60.96
	4	0.47	1.49	2.81	4.35	6.11
2.0	1	18.75	31.14	42.73	50.31	56.70
	2	0.40	1.24	2.34	3.62	5.09
	3	22.50	39.63	51.28	61.92	68.85
	4	0.65	1.99	3.75	5.81	8.14
3.0	1	21.12	36.33	48.10	63.40	67.45
	2	0.44	1.38	2.58	4.00	5.60
	3	25.35	44.59	60.12	73.15	79.50
	4	0.81	2.49	4.65	7.45	10.09
5.0	1	52.00	100.50	147.50	194.50	240.50
	2	2.32	7.08	13.07	20.28	28.16
	3	26.00	45.68	61.45	74.80	89.07
	4	1.00	3.03	5.62	8.69	12.12
∞	1	—	—	—	—	—
	2	—	—	—	—	—
	3	23.94	41.95	56.34	68.08	81.00
	4	1.07	3.24	6.01	9.37	12.95

The turning points in the curves for $M_t-\theta$ and $M_b-\phi$ are at initial cracking, yielding and ultimate conditions. The stiffness at these conditions are shown in **Table** 3-2.

$K_1=(M_{t.sy}/bd^2)/\theta_{sy}$: torsional stiffness till yielding

$K_2=(M_{b.sy}/bd^2)/\phi_{sy}$: bending stiffness till yielding

$K_3=(M_{t.cu}/bd^2)/\theta_{cu}$: torsional stiffness from yielding to failure

$K_4=(M_{b.cu}/bd^2)/\phi_{cu}$: bending siffness from yielding to failure

The relations between α and K_1 are shown in **Fig.** 3-16' and a variation of K_1 according to the combinations of h/b and α is as shown in **Fig.** 3-17. Considering the effect of M_b on torsional stiffness K_1, M_b does not influence in the sphere $0<\alpha\leqq1$. When α is larger than 1, M_b increases torsional stiffness for $h/b<1.6$ and on the other hand M_b decreases torsional stiffness for $h/b<1.6$.

The relations between α and K_3 are obtained as **Fig.** 3-18. The change of K_3 according to the combinatins of α and h/b is expressed in **Fig.** 3-19. Considering by this figure M_t decreases bending resistance when α is small and h/b is large.

With yieding, both bending and torsional stiffness drop remarkably. The values of K_2/K_1 and K_4/K_3

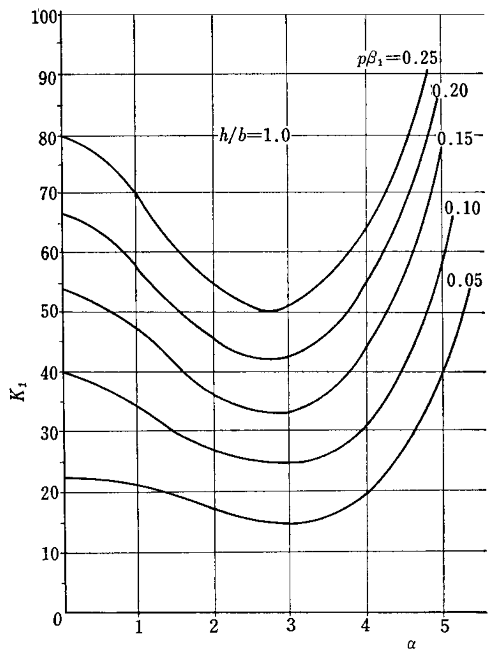


Fig. 3-16' $\alpha-K_1$

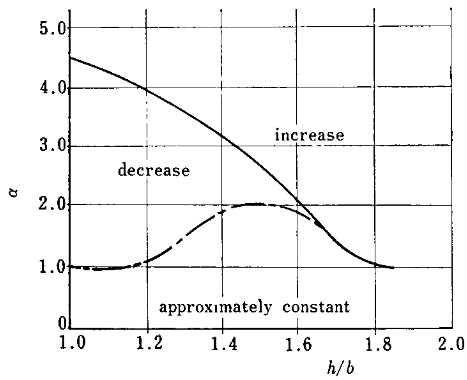


Fig. 3-17 variation of K_1

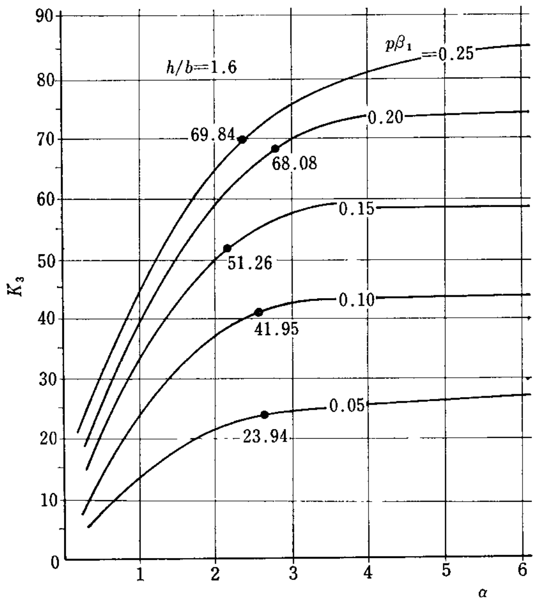


Fig. 3-18 $\alpha - K_3$

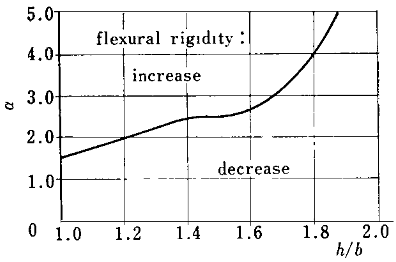


Fig. 3-19 variation of K_3

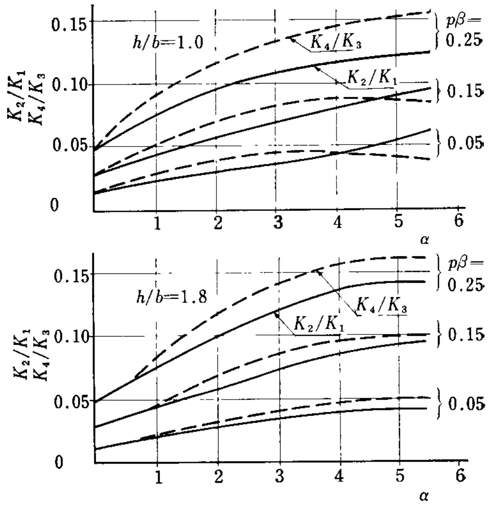


Fig. 3-20 $\alpha - K_2/K_1$, K_4/K_3

are calculated in **Fig.** 3-20. Increasing of steel ratio decreses the drop of stiffness. When α is small (M_t is large), drop of both bending and torsional stiffness is remarkable. When α is large, drop of stiffness becomes moderate and this inclination agrees with the testing results.

In spite of the values of α and h/b, drop of torsional stiffness is larger than the one of bending stiffness. This inclination becomes more remarkable according to increasing of the steel ratio.

3.5 Experiments

3.5.1 Tests and results

The summary of the tests and their results are shown in **Table** 3-3.

3.5.2 Reduction factor for strength

By checking the reliance for Exp. (3-13) with the experiments, the reduction factor for strength of the member is decided. The comparisons between the testing and calculating results of the yielding strength are shown in **Table** 3-4. Assuming that these ratios normally distributes, the lowest confidence limit is calculated in **Tabl.** 3-5. The strength reduction factor γ is decided equal to 0.75 for the risk of 4%.

3.6 Design method

It is proper to take the stregth criterion for design as yielding strenth. When the limitations are made for deformation or crack width, this citerion should be taken as the cracking strength. For the latter case, design method is described in Chp. 2.

3.6.1 Design formulas and charts

Table 3-3 Summary of the experiments

	No.		subject	numbers of specimens
basic experiment	I	1	pure torsion of plain mortor member	133
		2	pure torsion of plain concrete member with rectangular cross section	32
pure torsion and combined bending and torsion	II	1	combined bending and torsion of reinforced concrete member	18
		2	〃	8
		3	〃	14
		4	pure torsion of light weight concrete member with rect. cross section	20
		5	pure torsion of RC member with T-cross section	15
eccentric torsion	III	1	plain mortar member with rect. cross section	20
		2	RC member with rect. cross section	22
precast member	IV	1	conbined bending and torsion of precast jointed RC member	5
		2	pure torsion of precast jointed RC member	6
	V	1	dowel action in torsion	18
		2	torsional bond of steel	135

Table 3-4 test/calc. for $M_{t.sy}$

α	test/calc.						
0	0.80	0.83	1.11	1.06	0.81	0.81	0.81
	0.71	0.81	0.69	0.68	0.77	1.01	0.96
	0.96	0.96	1.07	1.10	1.01	0.91	0.78
	0.80	0.85	0.85				
0.5, 1.5	0.95	0.87	1.27	1.23	0.80	0.80	0.81
	1.00	0.85	1.09	0.98	1.03	1.12	
2.0, 3.0	0.76	0.81	0.61	1.18	1.48	0.81	0.93
	0.90						

Table 3-5 lowest conffidence limit

risk / α	0.03	0.05	0.07	0.10
0	0.82	0.83	0.83	0.84
0.5, 1.5	0.89	0.90	0.91	0.92
2.0, 3.0	0.74	0.76	0.77	0.79

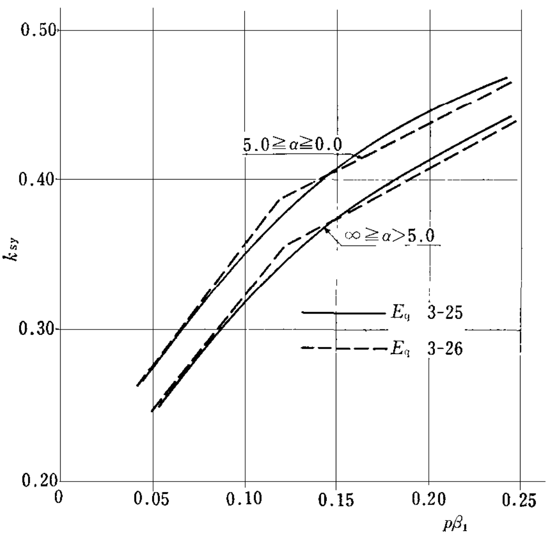


Fig. 3-21 $p\beta_1-k_{sy}$

The calculating formulas based on the mechanical analysis are complex and inconvenience for practice, so it is better to make these simple and to arrange the charts.

(1) Neutral axis coefficient

In Exp. (3-12) of k_{sy} assuming that $\varepsilon_{sy}=1430\times10^{-6}$ (SD 30) and $\varepsilon_{co}=3000\times10^{-6}$, the values of k_{sy} for various combinations of $p\beta$ and d/b can be calculated. These results show that the influence of d/b can be neglected and k_{sy} can be expressed as follows.

$0.000\leqq p\beta\leqq0.125$: $k_{sy}=0.193+1.610p\beta$ $\quad 0.0\leqq\alpha\leqq5.0$

$k_{sy}=0.16+1.610p\beta$ $\quad 5.0<\alpha\leqq\infty$

$0.125<p\beta\leqq0.250$: $k_{sy}=0.310+0.680p\beta$ $\quad 0.0\leqq\alpha\leqq5.0$

$k_{sy}=0.274+0.680p\beta$ $\quad 5.0<\alpha\leqq\infty$

About the relation of $p\beta$ and k_{sy}, the comparisons of strict and approximate calculating results are given in **Fig.** 3-21.

(2) Resisting moment coefficients

Using Exp. (3-13), calculating results are shown graphically in Fig. 3-15-(1)~(2) for the resisting moment.

(3) Deformation coefficients

The coefficients $C_{8\sim11}$ are shown in Fig. 3-12.

3.6.2 **Calculation of cross section**

When the followings are known or assumed which are M_t and M_b for the load, b for the cross section and σ_{co} and σ_{sy} for the materials, the cross section of the member may be calculated as follows.

The steel ratio $p\beta$ can be decided based on the deformability which is required for the whole arrangement of the member in the structure. According to the curve $p\beta - M_{t.sy}/bd^2\sigma_{co}$ (**Fig.** 3-22), d can be obtained.

The area of longitudinal bars is,

$$A_s = p\beta \frac{\sigma_{co}}{\sigma_{sy}} bd$$

About the stirrups, $r\beta_2$ is obtained from Fig. 3-14 and for this $r\beta_2$ the combination of a_{sy} and s are selected. The stirrups should enclose the longitudinal bars and be welded to them.

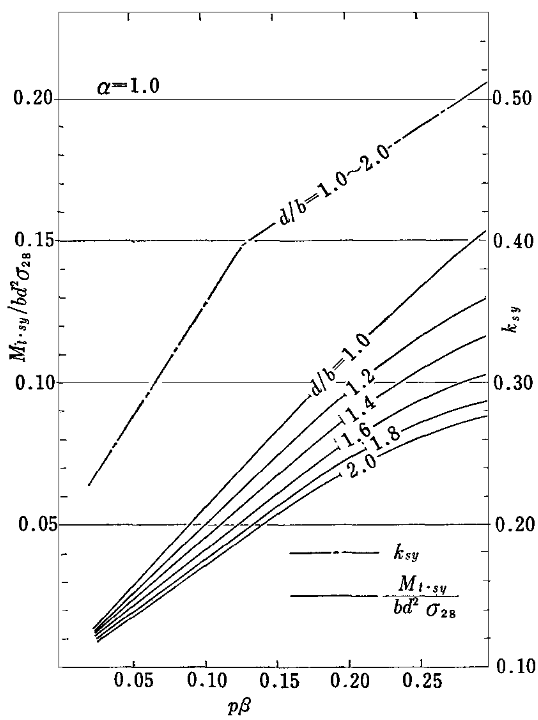


Fig. 3-22

3.6.3 **Calculating of deformation**

Deformation should be calculated to check the safety from the aspect of deformability and to know the secondary stress in the structure due to deformation. Using Exp. (3-8) for yielding and (3-11) for ultimate condition, deformation can be calculated.

3.6.4 **Checking of safety**

Safety of the member is considered in this proposed design method as follows.

1. Concrete strength for calculation is used as $\sigma_{co} = 0.85\sigma_{ck}$.
2. The yielding strength of the member is used for the criterion of design. When the member is reinforced suitably, there is some margin of strength from yielding to failure mainly because of the strain hardning of the steel.
3. Strength reduction factor γ (=0.75) is adapted.

In design, yielding and failure strength are calculated for the prearranged member and safety of strength is checked based on the design load. About doformation, the same check should be done. The graphs for α, d/b b, $p\beta$-$M_{t.cu}/bd^2\sigma_{co}$ are prepared as Fig. 3-11.

3.7 **Conclusions**

The failure mechanism of the reinforced concrete members under combined bending and torsion is made clear and the design method is proposed based on this mechanism. The member has the rectangular cross section and is reinforced by the longitudinal bars and the stirrups orthogonal to them within the balanced seel ratio. These steel is the mild steel with the clear yield point. The member fails by "skew bending" and torsional plastic hinge is formed at the same time as yielding of the tensile steel and it finishes with crushing of the compressing concrete.

The summaries of the conclusions are as follows.

(1) Hypothetical member

The failure surface of the member is a kind of the curved one and its idealized surface can be expressed as Fig. 3-1. For the purpose of analysis, the hypothetical member with this idealized surface is arranged.

(2) M_t and M_b acting on the actual member are translated to M_B and M_T acting on the hypothetical member, and these are shown in Fig. 3-4.

(3) The neutral axis and resisting moment coefficients of the hypothetical member are obtained based on the equilibrium and compatibity conditions about M_B and M_T. These coefficients are translated to the ones of the actual member as shown in Exp. (3-12) and (3-13). Relations of strain and stress for the steel and concrete are expressed in Exp. (3-2) and (3-3) especially emphasizing on near the ultimate conditions.

(4) Yield and ultmate strength of the member can be expessed by givening the strain conditions at each case to the above general formulas as in Exp. (3-13) and (3-15, 17, 19, 21)

(5) Deformation is expressed as twisting angle about the member's axis and rotation angle of the cross section about the axis orthogonal to the formor. Considering the length of the torsional plastic hinge, these angles can be obtained as Exp. (3-8) and (3-11).

(6) To make these formulas convenience for practice, the tables and charts are prepared from the results of calculation. For these calculations, ε_{co} is assumed to be 3000×10^{-6} and ε_{sy} to 1430×10^{-6} (SD 30).

For yielding k_{sy} and $M_{t.sy}/bd^2\sigma_{co}$ are shown in Fig. 3-21∼22. For failure k_{cu} and $M_{t.cu}/bd^2\sigma_{co}$ are shown in Fig. 3-10∼11.

About deformation θ_{sy}, ϕ_{sy} and θ_{cu}, ϕ_{cu} are shown in Fig. 3-13 for some examples.

(7) Reinforcement

The components of the tensile forces at the failure surface can be devided into two, one is along to the longitudinal bars and the other is to the stirrups. The propotion of the longitudinal bar ratio and stirrup ratio is decided from the condition that these both steel yield at the same time. The relation of $\alpha - r\beta_2/p\beta_1$ is shown graphically in Fig. 3-14.

The longitudinal bar ratio $p\beta_1$ is given by Exp. (3-25) and $p\beta_1 - M_{.sy}/bd^2\sigma_{co}$ graphes are shown in Fig. 3-22. The spacing of the stirrups is decided by Exp. (3-27) from the condition that stress of the stirrup does not exceed the stress of the longitudinal bars.

(8) Design method

When M_t and M_b for load, b for the cross section and σ_{co} and σ_{sy} for the materials can be assumed or known, the cross section of the member will be decided based on deformability required for the member which is the compenent of the structucture. About the selected member, safety should be checked for both strength and deformation.

4. Eccentric torsion

The surface of the member is warped by torsion and the secondary torsional moment results from the restraint of this warping. When torsion acts about the axis which is eccentric from the axis through the shear center of the cross section, secondary bending moment will be caused. These eccentric torsion often acts on the member which is the element of the structure. In this chapter, strength and deformation are examined in the case of eccentric torsion. Before ihitial cracking it is analyzed by the elasto-plastic torsional theory considering all the cross section effective. This results may be applied to the service load condition. After cracking the internal force distribution is the "double skew bending condition", and strength and deformation are estimated by the skew bending theory.

4.2 Notations

- M_t : torional moment about the proper torsional axis
- M_{te} : eccentric torsional moment
- M_b : external bending moment
- M_b' : secondary bending moment due to eccentricity of torsion and restraint of the cross section
- M_b'' : composed moment of M_b and M_b'
- α : M_b/M_{te}
- γ : eccentric torsional coefficient
- σ_b : modulus of rapture of concrete
- e : eccentric distance from torsional axis to

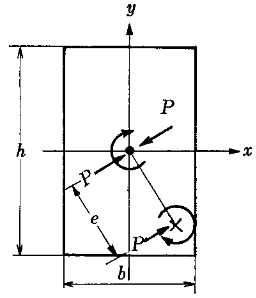


Fig. 4-1

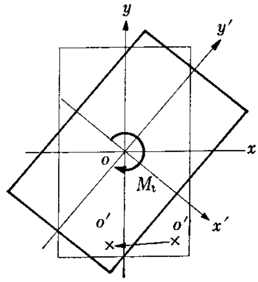


Fig. 4-2-(1)

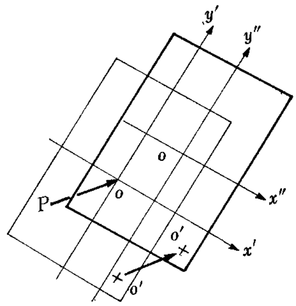


Fig. 4-2-(2)

shear center

4.3 Internal force

4.3.1 Intenal force due to eccentric torsion

The case is considered when one end of the member is fixed and eeeentric torsion acts to the other end. In **Fig.** 4-1, M_t about the shear center is,

$$M_t = M_{te} - P \cdot e \tag{4.1}$$

P can be decided based on the condition that the point O' does not move. As shown in **Fig.** 4-2-(1), the cross section rotates owing to M_t and then deflection δ_1 of O' is,

$$\delta_1 = e \cdot \tan \Theta \fallingdotseq e\Theta = e \cdot l\theta = el\frac{M_t}{k_2 b^3 hG} = el\frac{M_{te} - P_e}{k_2 b^3 hG}$$

Deflection δ_2 of O' due to P is as follows. In **Fig.** 4-2-(2), the deflection angle θ_{ol} at O is,

$$\theta_{o1} = \frac{Pl^2}{2EI}$$

$M_0(=Pl/2)$ causes defletion angle θ_{o2} at O,

$$\theta_{o2} = \frac{M_0 l}{EI}$$

The cross section including the point O is restrained and deflection angle is zero.

$$\theta_{o1} + \theta_{o2} = -\frac{Pl^2}{2EI} + \frac{M_0 l}{EI} = 0, \quad \delta_2 = \frac{Pl^3}{12EI}$$

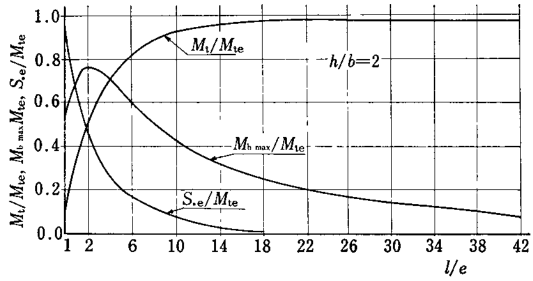


Fig. 4-3 $e/l - M_t/M_{te}$, $M_{b.\max}/M_{te}$, $S\,e/M_{te}$

$\delta_1 + \delta_2 = O$,

$$P = \frac{M_{te}}{el + \left(\frac{G}{E}\right)\left(k_1\frac{b^3 h}{12I}\right)\left(\frac{1}{e}\right)} = \frac{1}{1+\gamma} \cdot \frac{M_{te}}{e}$$

Then the internal forces due to M_{te} are obtained as follows.

$$\left.\begin{aligned} M_t &= M_{te} - Pe = \frac{\gamma}{1+\gamma} M_{te} = C_1 M_{te} \\ M_b' &= \frac{Pl}{2} = \frac{1}{2(1+\gamma)} \cdot \frac{l}{e} M_{te} = C_2 M_{te} \\ S &= \frac{1}{1+\gamma}\frac{M_{te}}{e} = C_3 \frac{1}{e} M_{te} \end{aligned}\right\} \tag{4-2}$$

$$\gamma = \left(\frac{G}{E}\right)\left(k_1\frac{b^3 h}{12I}\right)\left(\frac{l}{e}\right)^2, \quad I = I_x \sin^2\varphi' + I_y \cos^2\varphi'$$

From the calculating results using Exp. (4-2), some examples of M_t/M_{te}, $M_{b.\max}/M_{te}$ and $S \cdot e/M_{te}$ for the various values of l/e are shown in **Fig.** 4-3.

4.3.2 Internal force due to eccentric torsion and bending

The composed moment M_b'' of M_b and M_b' is obtained as follows. In **Fig.** 4-4, the x-directions and y-direction components of M_b' are $M_b' \sin\varphi'$ and $M_b' \cos\varphi'$ respectively, so M_b'' is,

$$M_b'' = \sqrt{(M_b + M_b' \sin\varphi')^2 + (M_b' \cos\varphi')^2} \tag{4-3}$$

$$\varphi'' = \frac{\pi}{2} - \tan^{-1}\frac{M_b' \cos\varphi'}{M_b + M_b' \sin\varphi'}$$

The internal forces due to M_{te} and M_b are M_t in

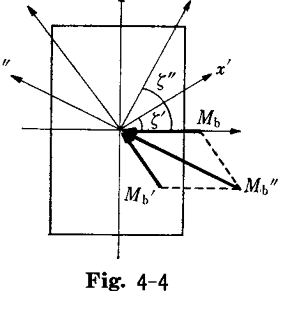


Fig. 4-4

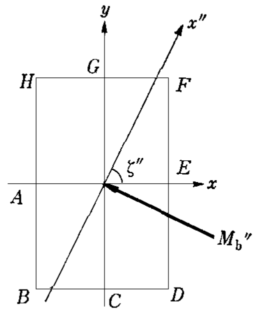


Fig. 4-5

Exp. (4-2) and M_b'' in Exp. (4.3).

4.4 Cracking strength

It cracks when the principal stress due to shearing stress of M_t and normal stress of M_b'' exceeds the modulus of rupture of concrete. The cracking strength is the smallest one in the followings, and in these formulas $A, B, C, \cdots H$ show the points of the cross section in **Fing.** 4-5.

$$\left.\begin{aligned}\frac{M_{te}(A,E)}{b^2h\sigma_b}&=\frac{A_1A_2}{\pm1+\sqrt{1+(A_2A_4)^2}}\\ \frac{M_{te}(B,F)}{b^2h\sigma_b}&=\pm\frac{A_1A_3}{1+\frac{b}{h}\cot\varphi''}\\ \frac{M_{te}(C,G)}{b^2h\sigma_b}&=\frac{2A_1A_3}{\pm1+\sqrt{1+(2K_3A_3A_4)^2}}\\ \frac{M_{te}(D,H)}{b^2h\sigma_b}&=\pm\frac{A_1A_3}{1-\frac{b}{h}\cot\varphi''}\end{aligned}\right\}\quad(4\text{-}4)$$

$$A_1=\frac{2(1+\gamma)\frac{e}{l}}{\sqrt{\left\{2\alpha(1+\gamma)\frac{e}{l}+\cos\varphi'\right\}^2+\sin^2\varphi'}}$$

$$A_2=\frac{4I}{b^3h}\sec\varphi'',\quad A_3=\frac{A_2}{2}\cdot\frac{b}{h}\cot\varphi'',$$

$$A_4=\frac{A_1}{k_1}\cdot\frac{\gamma}{1+\gamma}$$

4.5 Yielding and failure strength

After cracking of the member under pure torsion or combined bending and torsion, skew bending acts on this member. For the eccentric torsion it becomes "double skew bending condition". Strenth and deformation of the member are obtained from the equilibrium and compatibility conditions as discussed in Chap. 3.

4.6 Conclusions

When the torsional axis is eccentric from the shear center of the cross section and waping of the cross section is restrained, the secondary torsional and bending moments and shear force are introduced. These moment and force can not be neglected for the member having not enough long span.

(1) Before cracking

The intenal forces due to the eccentric torsion are M_t, M_b' and S as expressed in Exp. (4-2). The coefficient γ is the one about the property of materials, shape of the cross section and the ratio of span and eccentric distance.

For combined bending and torsion, M_b'' acts instead of M_b' which is the composed moment of external and secondary bending moment.

Cracking strength of such member can be obtained based on the principal stress and modulus of ruptupe of the concrete.

(2) After cracking

Based on the "double skew bending" concept, strength and deformation can be obtained.

5. Conclusions

In this paper the past reserches and design criterions about torsion of structural conrete are studied. Cracking and failure mechanism are analyzed and the design methods are proposed based on these analytical results.

(1) Torsional moment acts to the member often as the main load according to the member's arrangement in the structure, shape of the cross section and loading condition. Torsion acting to the reinforced concrete member changes greatly the equilibrium mechanism especially after cracking, and it is harmful to safety of the member from the aspects of strength and deformation.

(2) The categories for study of design method of torsion in the reinforced concrete members are divided broadly into three. They are truss analogy method, skew bending method and the method based on the experimental results.

(3) Generally in torsion, there is a little margin of strength from initial cracking to failure, and drop of torsional stiffness just after cracking is remarkably. If the member is reinforced not suitably, it fails sometimes at the same time as cracking. So, according to circumstances it is proper to design based on the cracking strength.

Based on the elasto-plastic condition of stress for torsion, cracking strength can be estimated exactly. The contribution of steel to strength is considered by the equivalent sectional area.

(4) After cracking, internal equilibrium mechanism of the member changes entirely and becomes skew bending condition. Based on the failure surface of the member under torsion, the hypothetical member is set up which is the double reinforced member having the inverted trapezoid cross section. On this member the skew bending moment acts, and from the equilibrium and compatibility conditions the general expressions about strength and deformation are derived as Eq. (3-4)～(3-11). Strength and deformation for the actual member at yielding and failure are expressed by givening the strain conditions to these general expressions as Eq. (3-13)～(3-23).

The reinforcing methods are also proposed, about the propotion of stirrup and longitudinal bar ratio as Eq. (3-24), longitudinal steel ratio as Eq. (3-25), maximum spacing of stirrups as Eq. (3-27).

From the calulating results using these expresstions, the charts and tables are prepared to make convenience for practical use. Many experiments of pure torsion and combined bending and torsion confirm these analysis and design methods.

6. Acknowledgement

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